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Letter to the Editor

Obtaining the static solution from the forced vibration situation in a common structural dynamics system

E. Alberdi[a,b], M. Alemany[a,b], P.A.A. Laura[a,b,*]

[a] Department of Engineering, Institute of Applied Mechanics (IMA), Universidad Nacional del Sur, Horneros 160, 8000 Bahia Blanca, Argentina
[b] Department of Physics, Institute of Applied Mechanics (IMA), Universidad Nacional del Sur, Horneros 160, 8000 Bahia Blanca, Argentina

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1. Introduction

The present study does possess only a didactic purpose. Consequently, no claim of originality is made. However, it is felt that it serves a very useful purpose since it shows that a basic, static solution may be obtained as a limiting case, for $\omega \to 0$, from the rather complicated dynamics solution when the structural system is subject to $P_0 \cos(\omega t)$, see Fig. 1(a) and (b), which depict the dynamic and static situations, respectively. It is assumed in both cases, that the beam–point mass arrangement is originally at equilibrium and that one is interested in obtaining the displacement and stress resultants under the action of $P_0 \cos(\omega t)$ (dynamic case) and P_0 for the static configuration.

The presence of the point mass M, instead, is taken into account when determining the natural frequencies of the overall system.

2. Discussion of both problems

Consider first the dynamic system depicted in Fig. 1(a). The steady state response is given by Ref. [1]:

$$w(x,t) = W(x)\cos(\omega t), \tag{1}$$

where

$$W(x) = -\beta_3\left(\sin kx - \sinh kx - \frac{\sin kL + \sinh kL}{\cos kL + \cosh kL}(\cos kx - \cosh kx)\right), \tag{2}$$

*Corresponding author. Department of Engineering, Institute of Applied Mechanics (IMA), Universidad Nacional del Sur, Horneros 160, 8000 Bahia Blanca, Argentina.
E-mail address: ima@criba.edu.ar (P.A.A. Laura).

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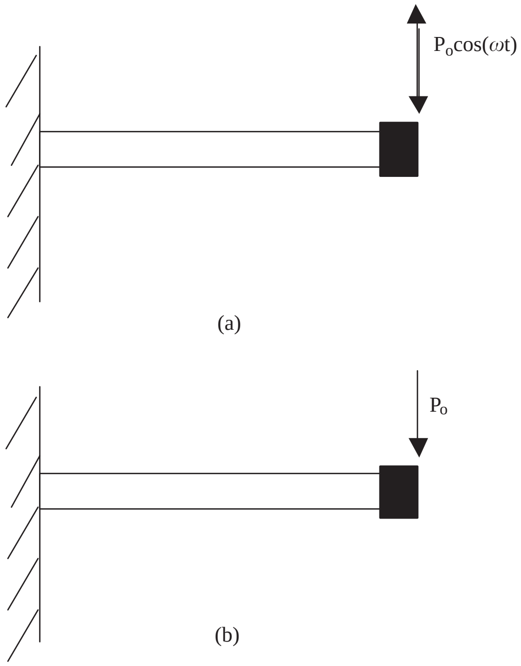

Fig. 1. (a) Forced vibration situation, (b) static problem.

$$m = \frac{M}{M_b}, \quad \rho A_0 L = M_b, \quad \text{beam mass}, \tag{3,4}$$

$$\beta_1 = \left(-\frac{\sin kL + \sinh kL}{\cos kL + \cosh kL}(\sin kL - \sinh kL) - \cos kL - \cosh kL \right), \tag{5}$$

$$\beta_2 = \left(\sin kL - \sinh kL - \frac{\sin kL + \sinh kL}{\cos kL + \cosh kL}(\cos kL - \cosh kL) \right), \tag{6}$$

$$\beta_3 = \frac{P_0 L^3/(EI)}{\beta_1 \alpha_1 (\omega/\omega_1)^{3/2} + \beta_2 m \alpha_1 (\omega/\omega_1)^2}, \tag{7}$$

$$\alpha_1 = kL\sqrt{\frac{\omega_1}{\omega}}, \quad \omega_1, \quad \text{fundamental circular frequency}. \tag{8,9}$$

On the other hand, the static solution of the problem shown in Fig. 1(b) is described by the elementary differential system:

$$EI\frac{\mathrm{d}^4 W_e(x)}{\mathrm{d}x^4} = 0, \tag{10}$$

$$W_e(0) = \frac{\mathrm{d}W_e}{\mathrm{d}x}(0) = 0, \tag{11}$$

$$\frac{d^2 W_e}{dx^2}(L) = 0, \tag{12}$$

$$-EI\frac{d^3 W_e}{dx^3}(L) = P_0, \tag{13}$$

whose solution is

$$W_e = \frac{1}{2}\frac{P_0 L^3}{EI}\left(-\frac{1}{3}\left(\frac{x}{L}\right)^3 + \left(\frac{x}{L}\right)^2\right). \tag{14}$$

One expands now $\sin kx, \cos kx, \sinh kx, \cosh kx, \sin kL, \cos kL, \sinh kL, \cosh kL$ in terms of kx and kL in Eq. (2) and obtains, taking the first terms:

$$\sin kx \simeq kx - \tfrac{1}{6}(kx)^3, \tag{15}$$

$$\sinh kx \simeq kx + \tfrac{1}{6}(kx)^3, \tag{16}$$

$$\cos kx \simeq 1 - \tfrac{1}{2}(kx)^2 + \tfrac{1}{24}(kx)^4, \tag{17}$$

$$\cosh kx \simeq 1 + \tfrac{1}{2}(kx)^2 + \tfrac{1}{24}(kx)^4. \tag{18}$$

One proceeds in the same fashion when expanding in terms of kL. Substituting Eqs. (15) through (18) in β_1 and β_2 yields

$$\beta_1 \simeq -2 + \tfrac{1}{4}(kL)^4, \quad \beta_2 \simeq \tfrac{2}{3}(kL)^3. \tag{19, 20}$$

Similarly,

$$\frac{\sin kL + \sinh kL}{\cos kL + \cosh kL} \simeq \frac{2kL}{2 + \frac{1}{12}(kL)^4} \simeq kL. \tag{21}$$

Thus, the coefficient of Eq. (2) can be approximated by

$$\beta_3 \simeq \frac{P_0 L^3}{EI}\frac{1}{2(-kL)^3 + \frac{1}{4}(kL)^7 + m\frac{2}{3}(kL)^7} \simeq -\frac{P_0 L^3}{2EI(kL)^3}. \tag{22}$$

On the other hand, the parentheses contained in Eq. (2) can be approximated by

$$\left(\sin kx - \sinh kx - \frac{\sin kL + \sinh kL}{\cos kL + \cosh kL}(\cos kx - \cosh kx)\right) \simeq \left(-\tfrac{1}{3}(kx)^3 + Lk^3x^2\right). \tag{23}$$

Using Eqs. (21) and (22) one obtains

$$\begin{aligned} W(x) &= -\beta_3\left(\sin kx - \sinh kx - \frac{\sin kL + \sinh kL}{\cos kL + \cosh kL}(\cos kx - \cosh kx)\right) \\ &\simeq \frac{P_0 L^3}{2EI(kL)^3}\left(-\tfrac{1}{3}(kx)^3 + Lk^3x^2\right). \end{aligned} \tag{24}$$

Now making $k = \sqrt{\rho A_0/EI}\sqrt{\omega}$ and taking the limit when $k \to 0$ one obtains

$$\lim_{k\to 0} W(x) = \frac{1}{2}\frac{P_0 L^3}{EI}\left[-\frac{1}{3}\left(\frac{x}{L}\right)^3 + \left(\frac{x}{L}\right)^2\right] = W_e(x). \tag{25}$$

It is felt that the present example shows that the static, ‘‘strength of materials’’ solution can be obtained, as a limiting case, from the forced vibrations situation contained in rather more advanced structures courses. Is is important to point out that Soedel's treatise [2] emphasizes this fact when dealing with plate and shell-like structures.

Acknowledgements

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